

1. (10 points) Show that the  $z$  component of angular momentum for a point particle

$$L_z = xp_y - yp_x \quad (1)$$

equals the following expression in spherical coordinates

$$L_z = mr^2\dot{\phi}\sin^2\theta \quad (2)$$

2. (10 points) Write down the *Hamiltonian* of a free particle in spherical coordinates

*Hints:* The energy of the system is

$$E = \frac{1}{2}m(\dot{r}^2 + r\dot{\theta}^2 + r^2\sin^2\theta\dot{\phi}^2) \quad (3)$$

- a) write out lagrangian  $\mathbf{L} = T - V$   
b) Calculate momenta by differentiating  $\mathbf{L}$  with respect to velocity

$$p_i(q_i, \dot{q}_i, t) = \frac{\partial \mathbf{L}}{\partial \dot{q}_i} \quad (4)$$

- c) express the velocity in terms of the momenta by inverting the above equations  
d) calculate *Hamiltonian* using the definition of the Legendre transformation

$$H = \sum_i \dot{q}_i p_i - \mathbf{L} \quad (5)$$

3. (10 points) What is Feynman's answer to his own question "If, in some cataclysm, all of scientific knowledge were to be destroyed, and only one sentence passed on to the next generations of creatures, what statement would contain the most information in the fewest words?". What is yours?
4. (10 points) What is the probability of number 1 appearing in the first 20 decimal points of  $\pi$  (hint <http://www.wolframalpha.com/> and input "first 20 digits of pi")

5. (10 points) What is your feedback for the class? Do you want more English or Chinese to be used in class? Anything that you are interested not covered by the class? Do you think the grading rules to be fair? Please share your thoughts.